Review on the PhD thesis for obtaining
the educational and scientific degree "doctor"

Author of the thesis: Dimitar Slavchev
Topic of the thesis: Composite numerical methods and scalable tile algorithms
by professor Dr Ivan Dimov Lirkov
Institute of Information and Communication Technologies, Bulgarian Academy of Sciences
in professional area 4.5 "Mathematics", scientific specialty "Computational Mathematics"

1. Relevance of the topic

   Block methods and algorithms for solving systems of linear algebraic equations with dense matrices are studied in the PhD thesis. The subject of this research are methods based on hierarchical semi-separable compression. These methods are used to solve elliptic and parabolic problems in the field of anomalous diffusion described by the integral Laplacian fractional formulation and discretized in space by the finite element method. I think that the relevance of the topic is indisputable.

2. Main objectives

   - Comparative analysis of the performance of block methods and algorithms for solving systems of linear algebraic equations with dense matrices.
   - Comparative analysis of the performance and parallel efficiency of software packages applying block LU factorization for solving systems of linear algebraic equations with dense matrices.
   - Analysis of the performance, parallel efficiency and accuracy of a method for approximating the solution of systems of linear algebraic equations based on hierarchical semi-separable compression (HSS) from the STRUMPACK software package for systems with appropriate matrix structure.
   - Development of reordering algorithms for the unknowns for systems of linear algebraic equations arising from discretization by finite element method of fractional diffusion in order to improve the efficiency of hierarchical semi-separable compression when applied on the stiffness matrix.

- Numerical solution of elliptic and parabolic problems in the field of fractional diffusion, modeled with the integral formulation of the fractional Laplacian and discretized by finite elements.

3. Overview of the content of the thesis

The thesis contains 140 pages, consists of an introduction, 4 chapters, a conclusion and a list of cited literature with 90 titles in Bulgarian and English. In the thesis 7 tables and 46 figures are included.

The Introduction formulates the goals of the dissertation, the methodology used for their implementation, the content is briefly presented.

The first chapter has an introductory character. It describes the block methods for solving systems of linear algebraic equations with dense matrices analyzed in the thesis, as well as main estimates of their computational complexity. Direct methods for solving systems of linear algebraic equations with dense matrices are described. A direct method of Gaussian elimination and LU factorization based on it are considered. Hierarchical methods for solving systems with structured dense or sparse matrices are described. Special attention is paid to the hierarchical semi-separable (HSS) compression of dense matrices and its implementation in the software package with free access STRUMPACK. The ULV-like factorization of the compressed matrix and the solution of the system with such a factorized matrix are described. The advantages of the HSS compression-based method leading to less computational complexity for tasks with a suitable output matrix structure are considered.

The second chapter is devoted to the laminar flow around Zhukovsky airfoils. The obtained system of linear algebraic equations with dense matrix is used for the comparative analysis of the considered block algorithms. The external boundary value problem for the Laplace equation is presented in integral form. To discretize the obtained integral equation, a method of collocation boundary elements is applied. Numerical experiments were performed on computer systems with shared and distributed memory. A comparative analysis of the performance and parallel efficiency of the considered high-performance software packages for solving systems of linear algebraic equations using block LU factorization using Intel Xeon processors and Intel Xeon Phi accelerators is made. The efficiency on computer systems with shared memory of a method based on hierarchical semi-separable compression was also studied. The results are compared with those of the most efficient software package using block LU factorization.

The third chapter examines methods for solving a two-dimensional boundary value

problem for anomalous diffusion. Anomalous diffusion is described by a Laplace fractional operator defined in integral form by the Ritz potential. The finite element method is used for discretization. The fractional Laplacian is a nonlocal operator, as a result of which the stiffness matrix of the system of linear algebraic equations is dense. The speed of the hierarchical solver depends strongly on the properties of the off-diagonal blocks of the matrix. Five methods for rearranging the unknowns are presented. The aim is to improve the structure of the matrix. Numerical experiments were performed on computer systems with shared memory. A comparative analysis of the speed and accuracy of the approximate solution of the problem with the hierarchical method implemented in the STRUMPACK package when varying the proposed rearrangements of the unknowns is presented. The efficiency of the hierarchical solver was analyzed in comparison with the solver from the MKL package.

In the fourth chapter methods for solving a parabolic problem with two-dimensional anomalous diffusion in space are studied. For the discretization in time an implicit Euler method with a constant time step and a diagonal concentration of the mass matrix is applied. Thus, the problem is reduced to solving a series of systems of linear algebraic equations with the same transition matrix and changing right hand sides during steps over time. This allows the factorization to be done once. Under these assumptions, the hierarchical method based on HSS compression has been shown to have advantages over block LU factorization. A comparative analysis of the direct solver implemented in the MKL package and the method using HSS compression from the STRUMPACK package was made. For the second method, the times for execution of the separate parts of the algorithm are analyzed. The obtained results confirm the advantages of the hierarchical method in numerical solution of the considered fractional diffusion parabolic equation.

At the end, a list of publications on the topic of the thesis and a list of conferences where the results are presented are attached.

4. Scientific and applied scientific activity

The main contributions in the thesis are in the field of analysis of the performance of algorithms for solving linear systems with dense matrices.

The performance of algorithms for solving systems obtained after applying the method of boundary elements for the boundary value problem for laminar flow around Zhukovsky airfoils has been studied. The computational complexity, parallel efficiency, and relative error of a hierarchical semi-separable compression (HSS) method were studied. The comparative analysis includes two types of dense matrices, which

were obtained after discretization with: a) method of boundary elements of a boundary value problem for laminar flow around Zhukovsky airfoils; b) finite element method for two-dimensional fractional diffusion boundary value problem. A comparative analysis of the solvers from the MKL and STRUMPACK is made. Characterization was obtained depending on the relative error threshold in HSS compression of the cases in which the hierarchical method has better speed. It is shown that for the task of Zhukovsky airfoils using the method of boundary elements, the sequential numbering of the nodes along the boundary of the profiles leads to a matrix with a structure suitable for HSS compression. This is not the case for the fractional diffusion boundary value problem, discretized by the finite element method. In order to improve the efficiency of the hierarchical semi-separable compression, five methods for rearranging the unknowns have been proposed and studied. For three of them new algorithms and software implementations are developed. The comparative analysis shows a significant improvement of the results when applying the methods of nested dissection and recursive bisection. A method, algorithm and software implementation for numerical solution of a parabolic equation with fractional diffusion operator in space have been developed. It has been proven that for this non-stationary task, the computational complexity of the separate parts of the algorithm creates conditions for the advantage of the hierarchical method based on HSS compression. For all sizes in space of the discretized problem, as well as for all variants of the relative error threshold, the variant of the program using the solver from the STRUMPACK package has better performance than the one using the MKL.

5. Publications on the topic of the thesis

   The thesis includes results published in 7 papers in impact-ranked issues, 6 of the papers have been published and 1 has been accepted for publication.

6. Abstract

   The abstract fully and accurately reflects the content of the thesis and the main contributions presented for defense.

7. Critical remarks

   The statement is clear and very well illustrated with specific examples. I have no significant critical remarks on the thesis. In my opinion, the Gaussian method is explained in too much detail in the first chapter, in addition there are erroneous indices on pages 23, 28, 32, there are inaccuracies in the formulas on page 33. In Fig. 2.20 it is not clear what t_q means. I also do not understand what a relative error of

6.74 in Table 3.1 and 8.9 in Table 3.2 means, I expected there to be some explanation for why the error is large. These remarks in no way diminish the results obtained by Dimitar Slavchev, which represent original contributions in the field of analysis the performance of algorithms for solving linear systems with dense matrices.

8. Conclusion

Considering the relevance of the topic and the importance of the scientific contributions, I believe that the thesis fully satisfies the requirements of ADASRB, Regulations for implementation of ADASRB, as well as the requirements in the regulations of BAS and IICT-BAS. I propose to Dimitar Slavchev to be given

the educational and scientific degree "doctor"

in professional area 4.5 "Mathematics", scientific specialty "Computational Mathematics".

На основание
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